SPRING 2025 MATH 540: QUIZ 4

Name:

1. State Euler's Product Formula. (3 points)

Solution. For $n \geq 1$, $\phi(n) = n \cdot \prod_{d|n}(1 - \frac{1}{p})$.

2. Verify Gauss's theorem for $n = 124$. (3 points)

Solution. We have to show $124 = \sum_{d|124} \phi(d)$. $124 = 2^2 \cdot 31$. The divisors of 124 are: 1, 2, 4, 31, 62, 124. We have:

$\phi(1) = 1, \phi(2) = 1, \phi(4) = 2, \phi(31) = 30, \phi(62) = \phi(2) \cdot \phi(31) = 1 \cdot 30 = 30, \phi(124) = \phi(4) \cdot \phi(31) = 2 \cdot 30 = 60.$

Adding we have $1 + 1 + 2 + 30 + 30 + 60 = 124$.

3. For $a, b > 0$ prove that if $a \mid b$, then $\phi(a) \mid \phi(b)$. Your may use any formulas proven in class. (4 points)

Solution. Since a divides b, any prime factor of a is a prime factors of b (by the Fundamental Theorem of Arithmetic). Write $b = p_1^{e_1} \cdots p_r^{e_r}$, with each $1 \leq e_i$. After re-ordering the primes of a, we may assume $a = p_1^{f_1} \cdots p_t^{f_t}$, with $t \leq r$ and $1 \leq f_i \leq e_i$, for $1 \leq i \leq t$. Thus,

$$\phi(a) = a \cdot (1 - \frac{1}{p_1}) \cdots (1 - \frac{1}{p_t})$$
$$\phi(b) = b \cdot (1 - \frac{1}{p_1}) \cdots (1 - \frac{1}{p_r}),$$

which shows $\phi(a)$ divides $\phi(b)$ since every term in the first equation divides the corresponding term in the second equation.